

Calculation Policy

Division

Models and Images

Counting apparatus

Arrays

100 squares

Number tracks

Numbered number lines

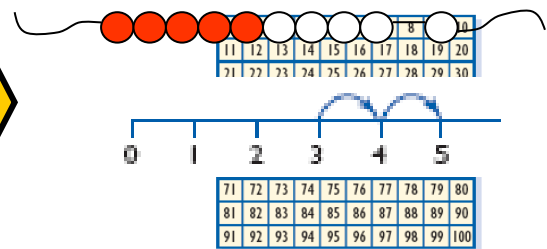
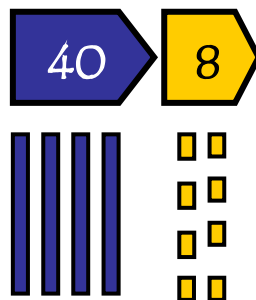
Marked but unnumbered lines

Empty number lines

Multiplication squares

Models and Images charts

ITPs – Multiplication remainders grid, Number Dials, Grouping



divided by group

into lots of \bullet into groups of

divisible — halve

half remainder

factor

Counting:

Year R (Early learning goal)

- Children count reliably with numbers from one to 20

Year 1

- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- count, read and write numbers to 100 in numerals, count in different multiples including 1s, 2s, 5s and 10s

Year 2

- count in steps of 2s, 3s, and 5s from 0, and count in 1s and 10s from any number, forward or backward

Year 3

- count forward and backward in multiples of 1s, 2s, 3s, 4s, 5s, 8s, 10s, 50s, and 100s;
- count up and down in 10ths, $\frac{1}{4}$ s and $\frac{1}{2}$ s - recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10

Year 4

- count forward and backward in multiples of 1-10s, 25s, 50s, 100s and 1000s;
- count backwards through zero to include negative numbers;
- count up and down in 10ths, 100ths, $\frac{1}{4}$ s, and $\frac{1}{2}$ s - recognise that hundredths arise when dividing an object by a hundred and dividing tenths by ten

Year 5

- count forward or backward in multiples of 1-10s, 25s, 50s, 100s, 250s, 1000s, 10 000s, 100 000s and into negative numbers;
- Count in 10ths, 100ths, $\frac{1}{4}$ s and $\frac{1}{2}$ s.

Year 6

- count forward or backward in multiples of 1-10s, 25s, 50s, 100s, 250s, 500s, 1000s, 10 000s, 100 000s and into negative numbers.
- Count in 10ths, 100ths, $\frac{1}{4}$ s, $\frac{1}{2}$ s and in decimals such as 0.1s, 0.01s, 0.2s, 0.25s, 0.5s.

Recall of times tables and its associated division facts:

Year 2: 2, 5 and 10

Year 3: 2, 3, 4, 5, 6, 8, 10

Year 4: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Year 5: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Year 6: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Use the following 100 squares to provide visual aid as children get used to the position of numbers in relation to each other in our number system when counting.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10

10	20	30	40	50	60	70	80	90	100
110	120	130	140	150	160	170	180	190	200
210	220	230	240	250	260	270	280	290	300
310	320	330	340	350	360	370	380	390	400
410	420	430	440	450	460	470	480	490	500
510	520	530	540	550	560	570	580	590	600
610	620	630	640	650	660	670	680	690	700
710	720	730	740	750	760	770	780	790	800
810	820	830	840	850	860	870	880	890	900
910	920	930	940	950	960	970	980	990	1000

100	200	300	400	500	600	700	800	900	1000
1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
2100	2200	2300	2400	2500	2600	2700	2800	2900	3000
3100	3200	3300	3400	3500	3600	3700	3800	3900	4000
4100	4200	4300	4400	4500	4600	4700	4800	4900	5000
5100	5200	5300	5400	5500	5600	5700	5800	5900	6000
6100	6200	6300	6400	6500	6600	6700	6800	6900	7000
7100	7200	7300	7400	7500	7600	7700	7800	7900	8000
8100	8200	8300	8400	8500	8600	8700	8800	8900	9000
9100	9200	9300	9400	9500	9600	9700	9800	9900	10000

Provide children with opportunities to investigate and discover the patterns on a multiplication square. Allow them to realise the commutative nature of multiplication and how division facts can be derived from known multiplication facts.

X	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Reception

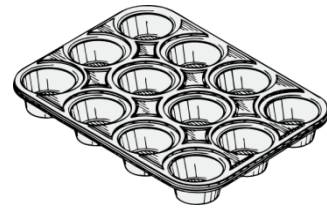
Early learning goal:

They solve problems, including doubling, halving and sharing.

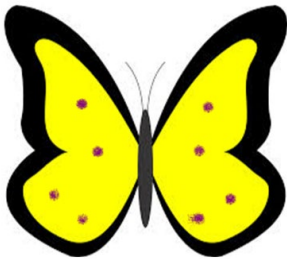
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical opportunities to share using a wide variety of equipment, counters, cubes, mini-bears etc.



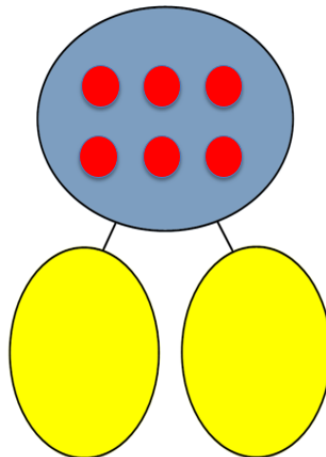
Children may also investigate by putting items into groups and arrays using egg boxes, ice cube trays and baking tins etc.



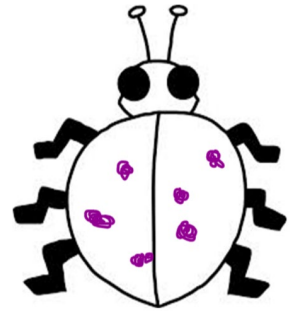
They may develop ways of recording calculations using pictures such as two sides of a lady-bird, butterfly.



One for you, one for me



6 shared between two is 3 each.

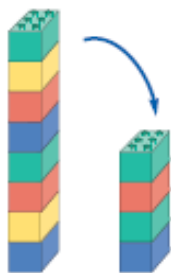


Year 1

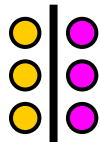
Counting choir

Count in steps of 2s, 5s and 10s forward and back from 0 and from any of its multiples using the 100 square and taking the opportunity to discuss patterns that are recognised.

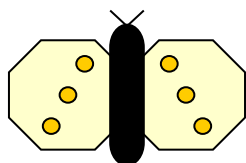
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



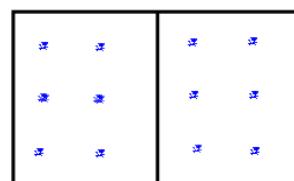
half of 8 is 4
 $8 \div 2 = 4$



Half of 6 is 3
 $\frac{1}{2}$ of 6 = 3



Half of 12. Share dots equally one by one.



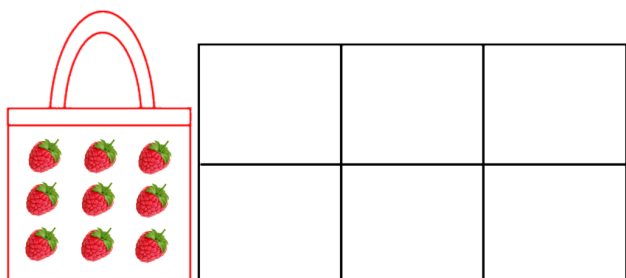
1. Division in the context of sharing:

1.1 Children learn to share objects practically in the context of the problem.



6 muffins shared between 3 people = 2 each
 $6 \div 2 = 3$

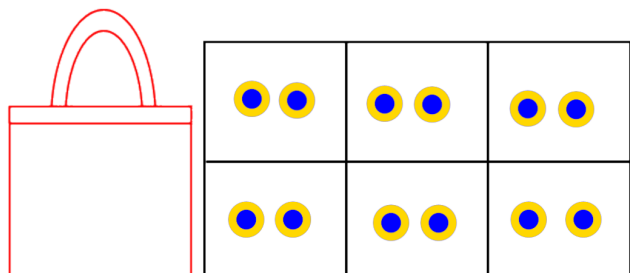
1.2 Children begin to share objects with the help of division sheets and investigate how they can make sure that the process is fair.



Share 9 raspberries equally between 3 children.

'What if the same number of raspberries is shared amongst 4 children?'

1.3 Children link their real life understanding to the mathematical process of sharing using cubes and begin to understand that the second number will tell them how many the items are shared between and the number of items in each box is the answer.



12 shared between 6 = 2
12 shared between 4 = 3
12 shared between 3 = 4
12 shared between 2 = 6

'I know that I'm correct because I checked that in each box there are equal amount of objects.'

2. Division in the context of grouping:

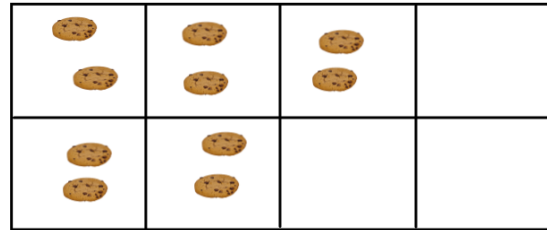
2.1 Children use grouping to solve problems involving division. With the help of division sheets children place the given number of objects into groups using the correct mathematical vocabulary. **Please note: to distinguish between grouping using multiplication and division a square based grouping sheet is used as shown below.**

10 cookies into groups of 2

How many groups?

$$10 \div 2 = 5$$

I have 10 cookies. Put them into groups of 2. How many groups have we got altogether?



2.2 Grouping with the use of jottings. Children first draw the total number of items using dots, then put circles around the given number of dots. They count the number of groups to obtain an answer.

I have 12 multilink cubes. If I put them into groups of 2, how many groups have I got?

12 **into groups of 2**

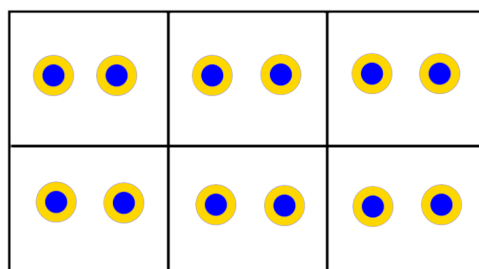
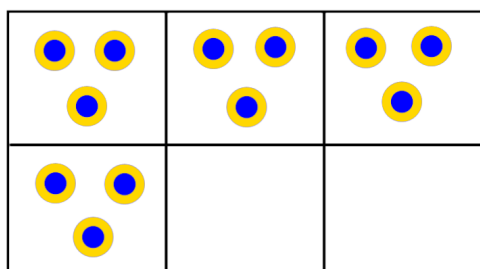
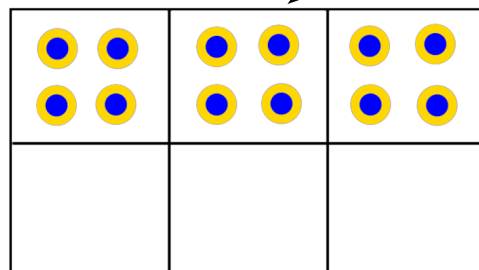
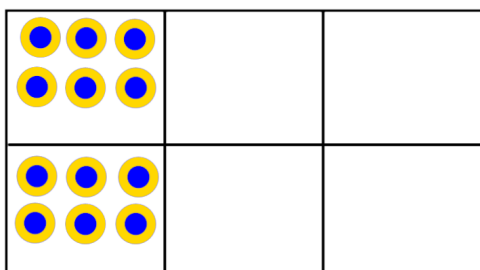
$$12 \div 2 = 6$$



2.3 Children should be given the opportunity to investigate the outcome of grouping the same number of objects into different sets and draw conclusions.

12 **into groups of 6** = 2
 12 **into groups of 4** = 3
 12 **into groups of 3** = 4
 12 **into groups of 2** = 6

'12 into groups of 4 gives me three groups.'

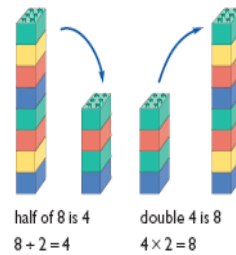
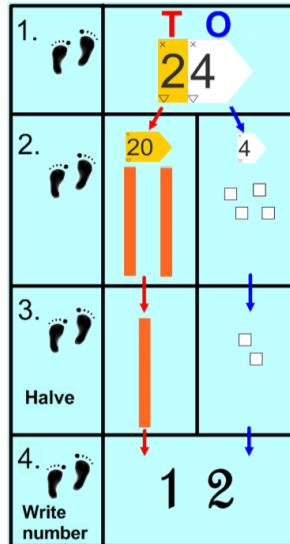
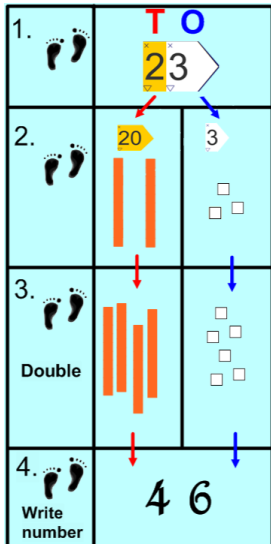


Year 2

Counting choir

Count in steps of 2s, 3s, 5s, 10s and 20s forward and back from 0 and from any of its multiples using the 100 / 200 square and taking the opportunity to discuss patterns that are recognised.

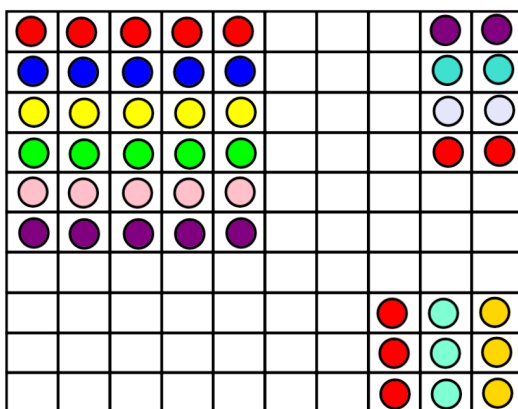
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200



Use fingers to work out doubles up to double 5.

Know doubles and corresponding halves and extend to partitioning numbers then double / partitioning numbers then halve.

Use known multiplication facts to work out corresponding division facts



Teaching should focus on the links made between multiplication and division to understand inverse operations.

... 'If I know that 4 rows of two = eight, I also know that eight into rows of 4 is 2 or eight into groups of 2 is 4!'

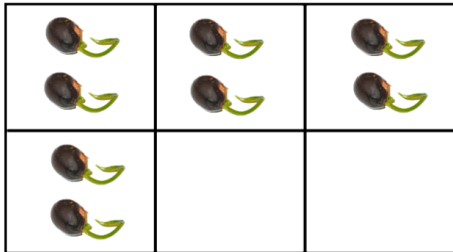
... 'If I know that 12 into groups of two = six, I also know that six groups of 2 is 12!'

12 into groups of 2
 $12 \div 2 = 6$

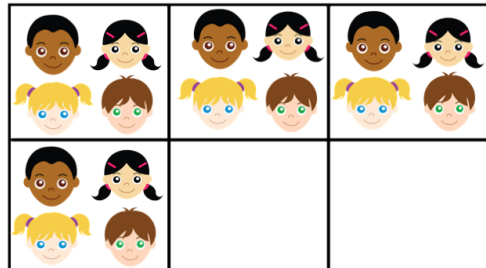


1. Children continue to use concrete materials and physical resources to share and group objects equally with or without the help of the laminated division sheets and link both these processes to the operation of division as the problems become more and more sophisticated.

Amy has 8 seeds and 4 pots. She puts the same number of seeds in each pot. How many seeds are in each pot?
8 seeds shared between 4 = 2 seeds each
 $8 \div 4 = 2$

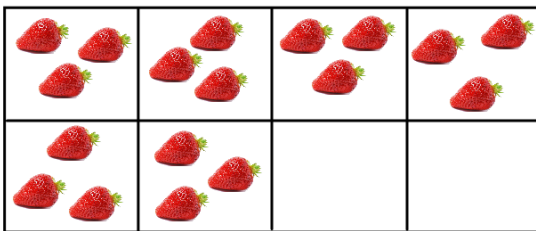


Mrs Jones puts 16 children into groups of 4 during a P.E. lesson. How many groups has she got?
16 children into groups of 4 = 4 groups
 $16 \div 4 = 4$

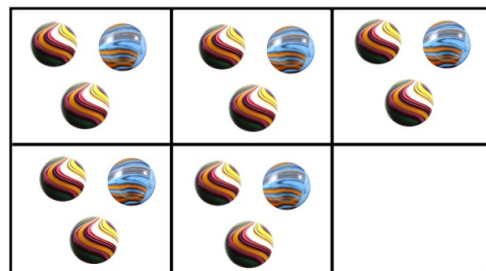


I have 18 strawberries. Put them into groups of 3. How many groups have we got altogether?

18 strawberries into groups of 3
How many groups?
 $18 \div 3 = 6$



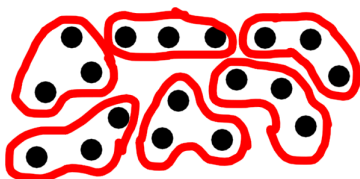
Some children share 15 marbles. They get 3 marbles each. How many children are there?
15 marbles into groups of 3 = 5 children
 $15 \div 3 = 5$



2. Grouping with the use of jottings. Children first draw the total number of items using dots, then put circles around the given number of dots. They count the number of groups to obtain an answer. In year 2 children are exposed to grouping in all multiples between 2-9.

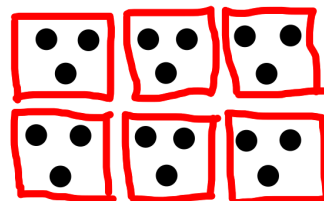
I have 18 multilink cubes. If I put them into groups of 3, how many groups have I got?

18 into groups of 3 = 6
groups $18 \div 3 = 6$ groups



Children who have secure knowledge of number clusters should arrange their dots using these.

18 into groups of 3 = 6 groups
 $18 \div 3 = 6$ groups

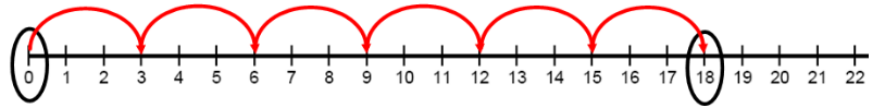


3. Children begin to use a marked number line to solve division problems.

Please note: Although division is associated with repeated subtraction, we teach children to **COUNT ON** in jumps of the given number because when it comes to teaching division with remainders on a number line, this is the only way it will work. This eliminates confusion.

This method requires children to find out how many jumps of 3 can they make between 0 and 18. They circle 0 and 18 on the number line before they commence their equal jumps of 3. The constant re-enforcement of vocabulary 'into groups of 3' is very important.

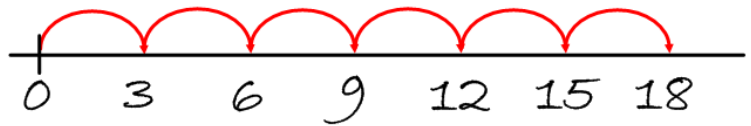
18 into groups of 3 = 6 groups
 18 into jumps of 3 = 6 jumps
 $18 \div 3 = 6$



4. More confident children, who are able to reliably count in multiples of 2, 3, 5, 10 use an empty number line to make their jumps.

They write their own numbers underneath the number-line each time they complete a jump to keep track of where they are. The challenge in this process is to remember to stop once they got to the required number. In this case, 18.

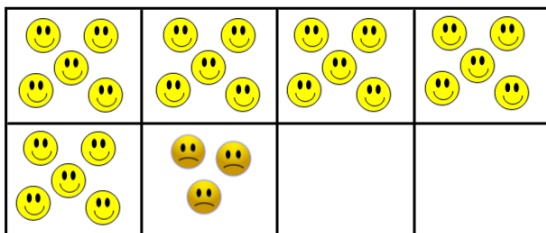
18 into groups of 3 = 6 groups
 18 into jumps of 3 = 6 jumps
 $18 \div 3 = 6$



5. Reinforce division as grouping through arrays and jottings and introduce remainders.

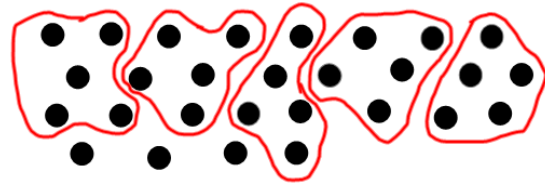
5.1 Practical using division mat.

28 children into groups of 5
 How many children left without a group?
 $28 \div 5 = 5 \text{ r } 3$



5.2 Jottings without the use of number clusters.

28 children into groups of 5
 How many children left without a group?
 $28 \div 5 = 5 \text{ r } 3$



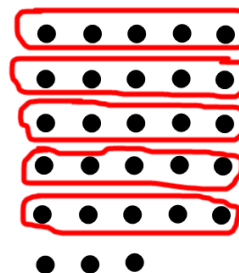
5.3 Jottings with the use of number clusters.

28 children into groups of 5
 How many children left without a group?
 $28 \div 5 = 5 \text{ r } 3$



5.4 Jottings using arrays.

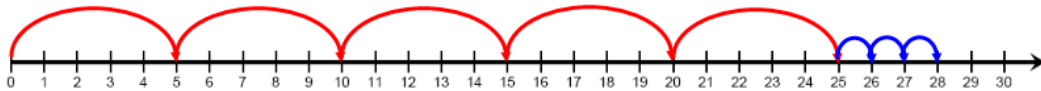
28 children into groups of 5
 How many children left without a group?
 $28 \div 5 = 5 \text{ r } 3$



6. Children begin to use a marked number line to solve division problems involving remainders.

The challenge in this process is to remember to not to carry on jumping in multiples of 5s after number 25 and to realise that the remaining jumps need to be made in jumps of ones to work out the remainder. Children must be able to count in multiples of 2, 3, 5 and 10 securely in order to use this method successfully.

28 children into groups of 5
How many children left without a group?
 $28 \div 5 = 5 \text{ r } 3$



Year 3, 4, 5 and 6: Short division

Children must have secure division facts knowledge to 10 x 10 in order for them to see the benefits of this quick efficient method. Understanding of grouping as a division concept is a vital skill for children to be successful users of short division.

Because children have to get used to a new layout, it is essential that the division method is taught on split screen, which models the use of concrete apparatus alongside the procedural method to support conceptual understanding.

Lesson design should incorporate the skill progression and use procedural variation as questions are introduced and get progressively more complex. The examples given below, follow this rule of sequence.

1) Division without remainder. Children use the Place Value grid to group Dienes.

Step 1.

Tens	Ones

 $2 \overline{) 62}$

Step 2.

Tens	Ones

 $2 \overline{) 62}$

Step 3.

Tens	Ones

 $2 \overline{) 62} \begin{matrix} 31 \end{matrix}$

2) Division with remainder in the Ones column.

Step 1.

Tens	Ones

 $2 \overline{) 65}$

Step 3.

Tens	Ones

 $2 \overline{) 65} \begin{matrix} 32r1 \end{matrix}$

Step 2.

Tens	Ones

 $2 \overline{) 65} \begin{matrix} 3 \end{matrix}$

Suggested mental maths starter before teaching the division method with remainders is to find remainders when dividing numbers mentally.
Example: $27 \div 5 = 5 \text{ r } 2$ or $38 \div 6 = 6 \text{ r } 2$ or $82 \div 9 = 9 \text{ r } 1$
All children should be able to calculate using this method by the end of year 3.

3) Division with remainder in the Tens and Ones column.

Step 1.

Tens	Ones

$$2 \overline{) 75}$$

Step 2.

Tens	Ones

$$2 \overline{) 75} \begin{matrix} 3 \\ \hline \end{matrix}$$

Step 3.

Tens	Ones

$$2 \overline{) 75} \begin{matrix} 3 \\ \hline 15 \end{matrix}$$

Step 4.

Tens	Ones

$$2 \overline{) 75} \begin{matrix} 37 \\ \hline 15 \end{matrix} r1$$

Once children understood the concept of division using Dienes, use PV counters to deepen understanding.

$$3 \overline{) 965}$$

Step 1.

Hundreds	Tens	Ones

$$3 \overline{) 965}$$

Step 2.

Hundreds	Tens	Ones

$$3 \overline{) 965} \begin{matrix} 32 \\ \hline \end{matrix}$$

Step 3.

Hundreds	Tens	Ones

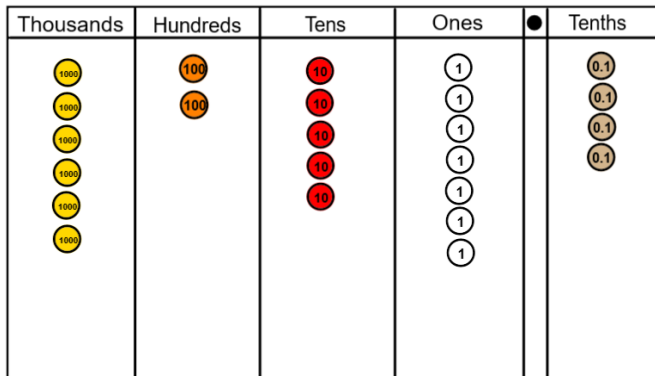
$$3 \overline{) 965} \begin{matrix} 321 \\ \hline 2 \end{matrix} r2$$

Step 4.

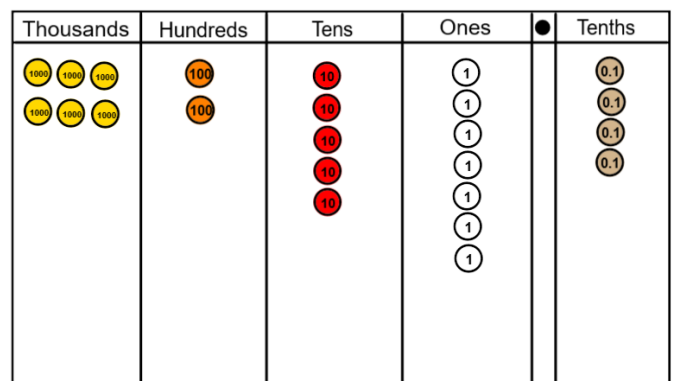
Hundreds	Tens	Ones

Extend this knowledge to much larger numbers which also includes decimals.

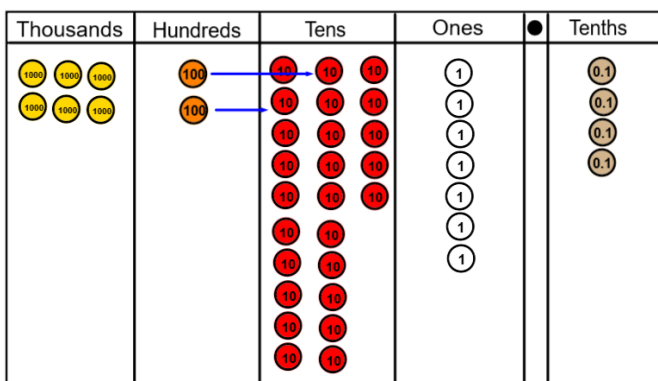
Step 1. $3 \overline{) 6257.4}$



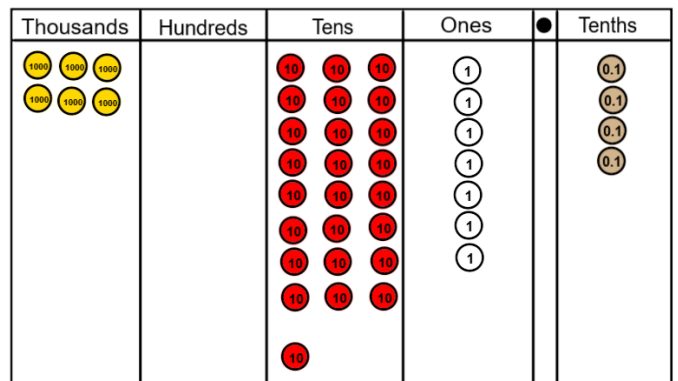
Step 2. $3 \overline{) 6257.4}^2$



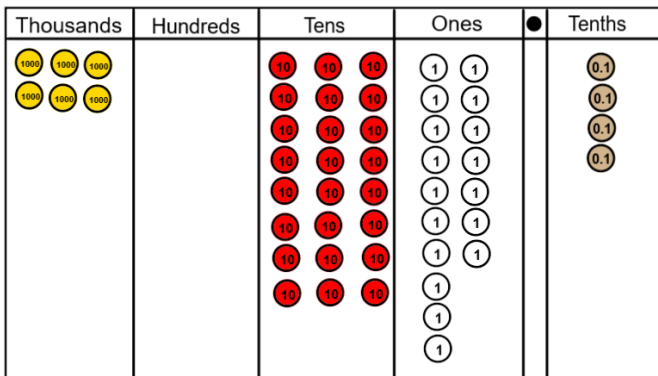
Step 3. $3 \overline{) 6257.4}^{20}$



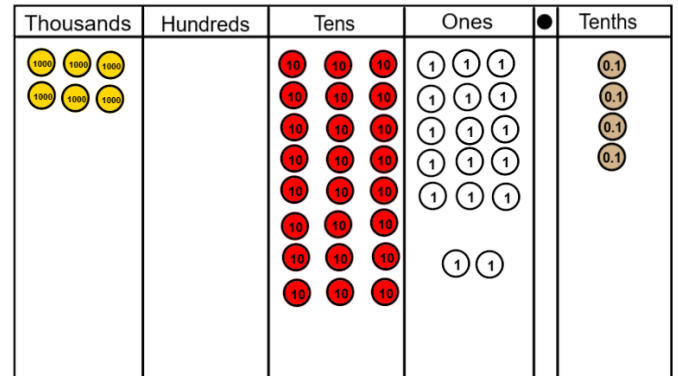
Step 4. $3 \overline{) 6257.4}^{208}$



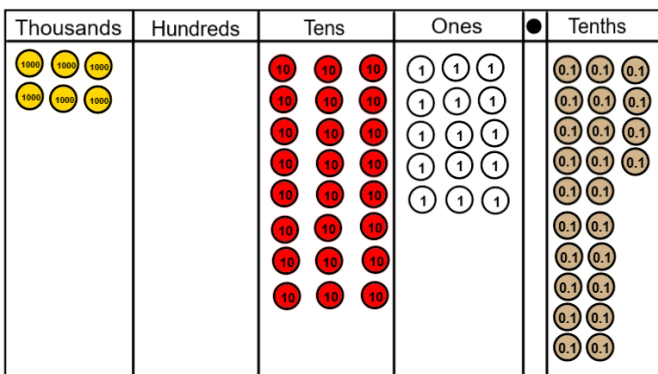
Step 5. $3 \overline{) 6257.4}^{2085}$



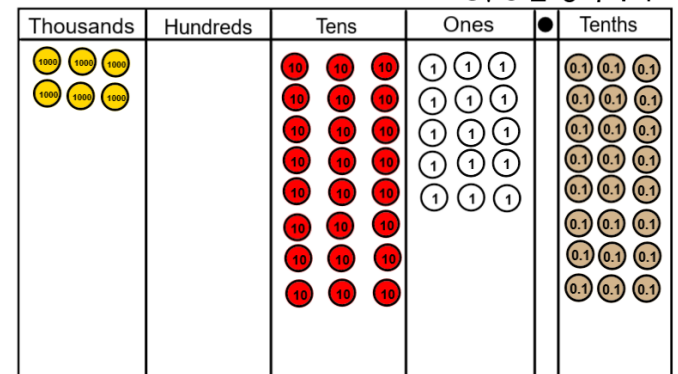
Step 6. $3 \overline{) 6257.4}^{208524}$



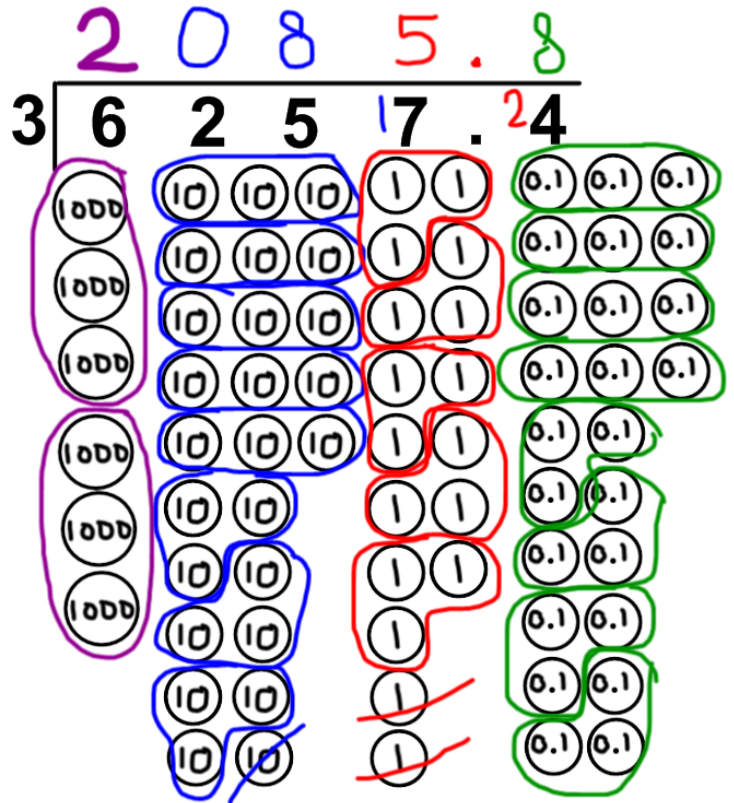
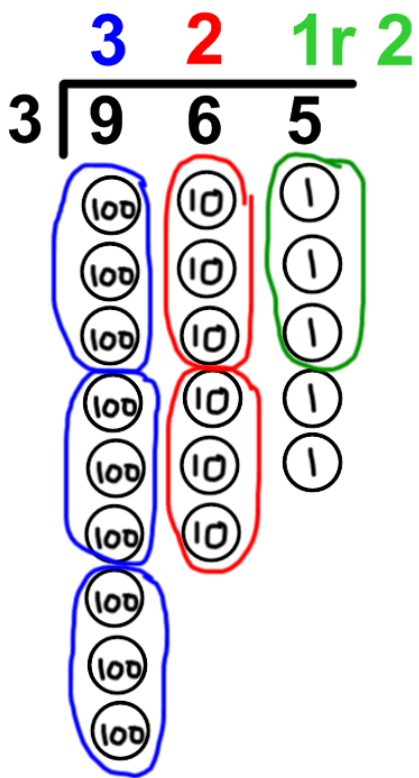
Step 7. $3 \overline{) 6257.4}^{208524}$



Step 8. $3 \overline{) 6257.4}^{2085.8}$



If children are confident with this method using the PV counters they should show understanding through pictorial representations. If they can do this once correctly, they can move onto calculating using numerals only.



Year 5 and 6: Long division HTU ÷ TU

This method is followed on from the short division, however uses a different format to make finding the remainder easier to calculate. Please note that it is not practical to use PV counters with long division. Children must be proficient with short division before moving onto long division.

When we first begin teaching this, provide children with an already prepared fact box. Once more confident, get children to create their own.

$$\begin{array}{r}
 0 \ 2 \ 4 \ r \ 1 \ 2 \\
 2 \ 4 \overline{) 5 \ 8 \ 8} \\
 \underline{- \ 4 \ 8} \\
 1 \ 0 \ 8 \\
 \underline{- \ 9 \ 6} \\
 1 \ 2
 \end{array}$$

Fact box:

2 x 24 = 48
3 x 24 = 72
4 x 24 = 96
5 x 24 = 120
6 x 24 = 144
7 x 24 = 168
8 x 24 = 192
9 x 24 = 216
10 x 24 = 240

An alternative method to the long division is to write remainder just like in short division. Children must ensure that their digits are written with enough gap in between them in order to be able to fit in the remainder.

$$\begin{array}{r}
 0 \ 2 \ 4 \ r \ 1 \ 2 \\
 2 \ 4 \overline{) 5 \ 8 \ 8}
 \end{array}$$

Children must be taught to express long division as remainders, decimals as well as mixed number fraction.

To express remainders as a decimal number, we must carry on with the division by bringing down a zero until we have remainders.

Children should use their knowledge of place value and conversions between fractions and decimals to express the answer as a decimal as well as a mixed number fraction.

$$\begin{array}{r}
 0 \ 2 \ 4 \ . \ 5 = 2 \ 4 \ \frac{1}{2} \\
 2 \ 4 \overline{) 5 \ 8 \ 8} \\
 \underline{- \ 4 \ 8} \\
 1 \ 0 \ 8 \\
 \underline{- \ 9 \ 6} \\
 1 \ 2 \ 0 \\
 \underline{- \ 1 \ 2 \ 0} \\
 0 \ 0 \ 0
 \end{array}$$

$$\begin{array}{r}
 0 \ 2 \ 4 \ . \ 5 = 2 \ 4 \ \frac{1}{2} \\
 2 \ 4 \overline{) 5 \ 8 \ 8 \ 120}
 \end{array}$$

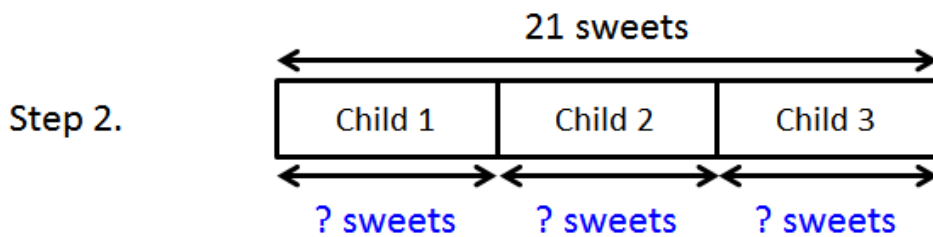
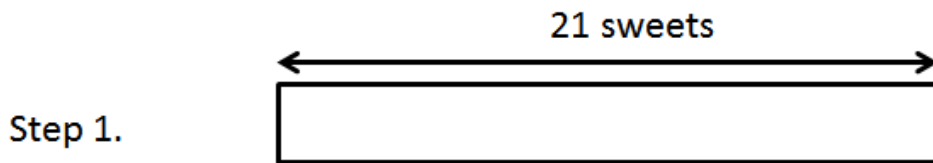
In the alternative method the zero is written next to the last digit of the dividend and carry the remainder in front of that zero.

In both of the above methods children should check if their answer is correct using inverse operations by multiplying their answer by the divisor and adding the remainder to their answer.

Using and applying: Once confident with this method, provide children with plenty of opportunities to be able to use and apply their newly gained skills to solve problems that involves getting answers with remainders and decimals.

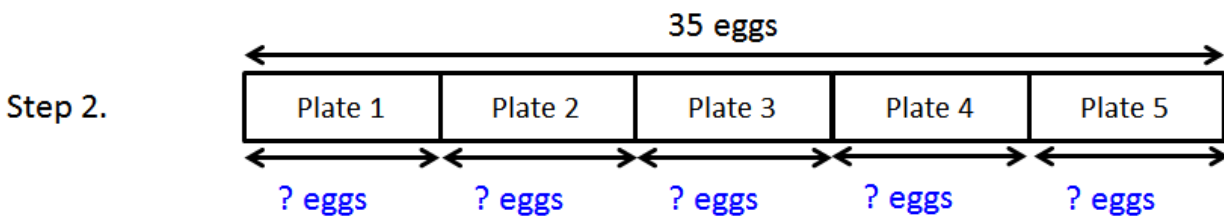
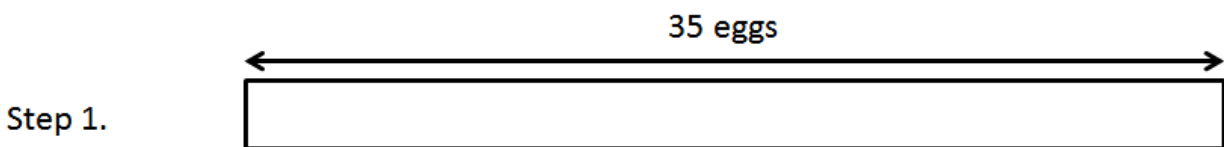
Problem solving involving multiplication using the bar model method

Mrs Morgan buys 24 sweets. She shares these equally amongst her 3 children. How many sweets do they each get?



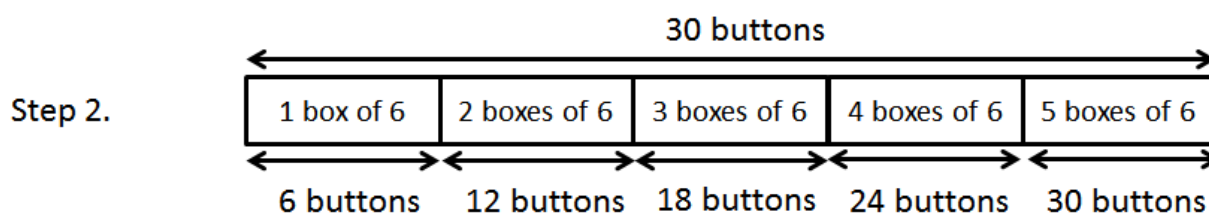
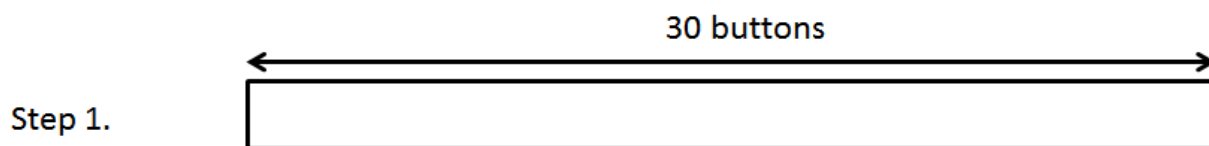
Step 3. **21 shared between 3 = 7**
 $21 \div 3 = 7$

A hotel chef fries 35 eggs for breakfast in a hotel.
He divides them equally among 5 plates.
How many eggs are on each plate?



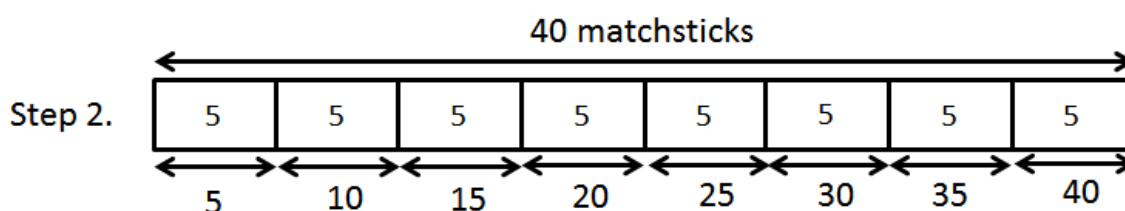
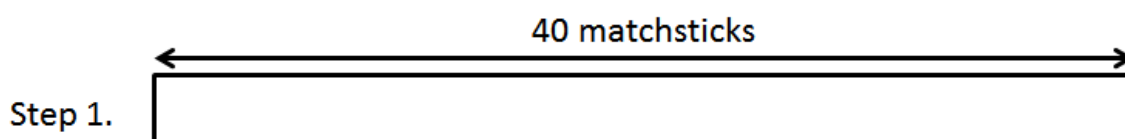
Step 3. **35 shared between 5 plates = 7**
 $35 \div 5 = 7$

Janet collects buttons and she has 30.
 She arranges them into boxes so there are 6 buttons in each box.
 How many boxes does she need?



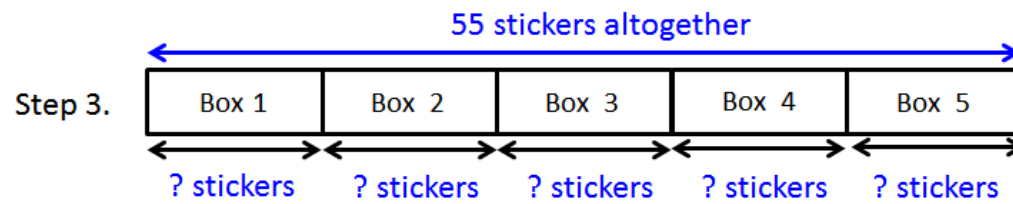
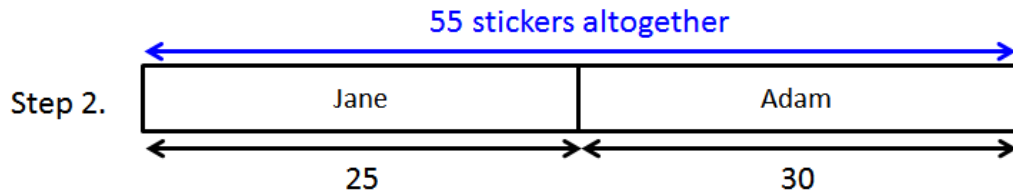
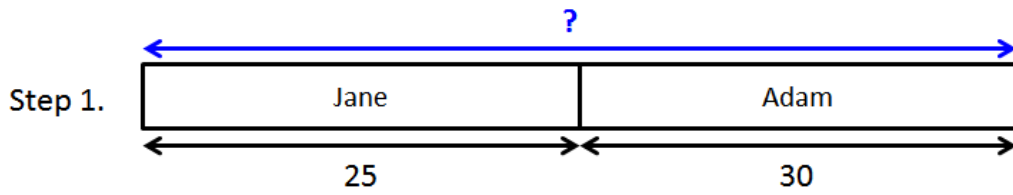
Step 3. **30 into boxes of 6 = 5 boxes**
 $30 \div 6 = 5$

Talia uses 40 matchsticks to make pentagons.
 Each pentagon is made with 5 matchsticks.
 How many pentagons does Talia make?



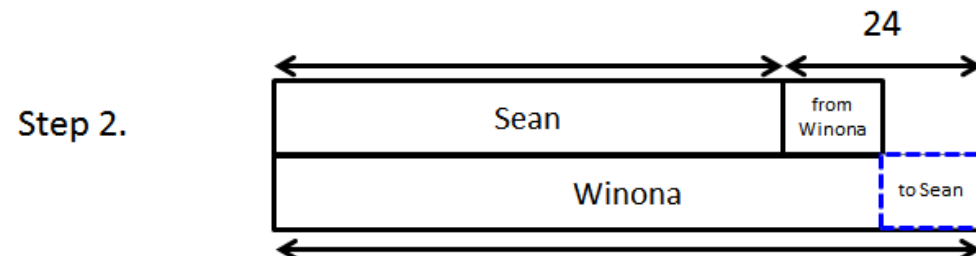
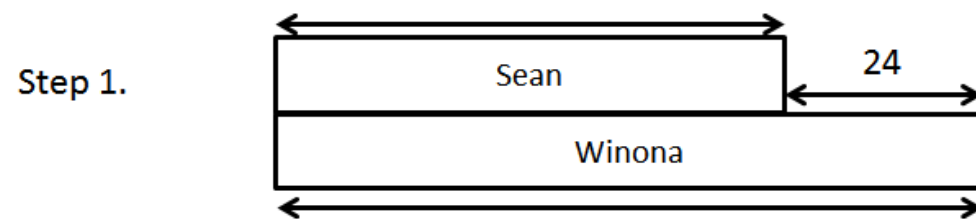
Step 3. **40 into groups of 5 = 8 pentagons**
 $40 \div 5 = 8$

Jane has 25 stickers. Adam has 30 stickers.
 They put all of their stickers equally into 5 boxes.
 How many stickers does each box have?



Step 4. **55 stickers shared between 5 boxes = 11 stickers**
 $55 \div 5 = 11$

Sean has 24 fewer toys than Winona.
 After Winona gives some toys to Sean, both of them have
 the same number of toys.
 How many toys does Winona give Sean?



Step 3. **$24 \div 2 = 12$**